The static potential beyond screening in the 3d SU(2) Higgs model

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The static potential in the 3d SU(2) Higgs model is computed by a variational calculation employing Wilson loops and two-meson operators. String breaking is demonstrated numerically, the breaking scale is determined and the results are compared with a quenched calculation.

#### 1. INTRODUCTION

A fundamental property distinguishing full from quenched QCD is the phenomenon of string breaking. Wilson loop (WL) calculations have well established linear confinement at intermediate distances, i.e. the potential of two static colour charges in the fundamental representation bound by a gauge string rises linearly with their separation r. For pure gauge theories this linear rise continues to infinity. In the presence of matter fields the string is expected to break when its energy is large enough to pair produce dynamical particles, which are then bound to the sources forming two static-light "mesons". The potential beyond the breaking scale  $r_b$  then saturates at twice the meson energy. The scale  $r_b$ can be estimated by equating the linear potential extracted from WL's with twice the meson energy. Although present day QCD simulations have reached such distances, direct evidence for string breaking and the saturation of the potential is not observed [1].

The problem is generic also for other theories with confinement. String breaking should occur in the confinement phase of the SU(2) Higgs model [2] and in the adjoint potential in pure gauge theory, but similarly this is not apparent in WL calculations [3]. This observation is paralleled in 3d gauge theories with and without matter fields [4], which qualitatively have the same confinement and screening properties as in 4d.

The failure of WL calculations as well as a recent strong coupling analysis [5] suggest that WL's have poor overlap with the two-meson (MM) states, and hence MM operators and their mixing with WL's should be considered. In view of the qualitative similarity between 3d and 4d gauge theories, it seems expedient to perform such an analysis in a simpler theory, the confinement phase of the 3d SU(2) Higgs model. Besides having bosonic fields only and being able to simulate larger volumes, the superrenormalisability of 3d gauge theories accounts for analytically known constant physics curves [6]. Moreover, the mass spectrum in the confinement phase of the model is well known and exhibits very good scaling behaviour [7,8]. Working at the same point in the confinement phase and with the notation as in [7,8], we approach the continuum by simulating three gauge couplings,  $\beta_G = 5.0, 7.0, 9.0$ , while keeping the continuum scalar self-coupling fixed at  $\beta_G \beta_R / \beta_H^2 = 0.0239$  (for more details see [9,7]).

# 2. MIXING ANALYSIS

The standard operator to compute the static potential is the WL of area  $r \times t$ , corresponding to a correlation  $G_{SS}(t)$  of a string of length r, Fig. 1. An operator that should be well suited to describe the MM state after string breaking is  $G_{MM}(t)$ . In order to check that this is indeed the case, we have also measured the correlator of one static-light meson, given by  $G_{M}(t)$ . The transition from the string to the MM state is represented by  $G_{SM}$ .

The static potential is now studied by finding

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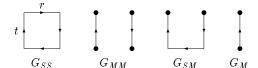


Figure 1. Operators used to extract the static potential. The dots represent scalar fields.

the lowest energy eigenstate contributing to the matrix correlator at every given r,

$$G(r,t) = \begin{pmatrix} G_{SS}(r,t) & G_{SM}(r,t) \\ G_{MS}(r,t) & G_{MM}(r,t) \end{pmatrix}. \tag{1}$$

This strategy has also been employed in calculations of the adjoint potential [3].

In order to enhance the signal, we have used smeared field variables obtained by the standard "fuzzing" algorithm [10] for the links and a suitable modification of scalar smearing described in [7]. For the operator basis we selected three link and five scalar fuzzing levels, leaving us with an  $8 \times 8$  correlation matrix  $G_{ik}(r,t) = \langle \phi_i \phi_k \rangle$ , where the  $\phi_i$  stand for string or MM operators at a given fuzzing level. The procedure to diagonalise  $G_{ik}$  by a variational method has been discussed in detail in [3,7]. It results in eigenvectors  $\Phi_i = \sum_k a_{ik} \phi_k$  from which the correlation functions of the (approximate) eigenstates of the Hamiltonian may be calculated.

$$\Gamma_i(t) = \langle \Phi_i(t)\Phi_i(0)\rangle = \sum_{j,k=1}^8 a_{ij}a_{ik} G_{jk}(t).$$
 (2)

The coefficients  $a_{ik}$  are a measure for the overlap between the operators used in the simulations and the energy eigenstates. This analysis not only allows to extract the ground state energy, but also those of the first excited states as well as the "composition" of the mass eigenstates in terms of the original operator basis.

### 3. RESULTS

In Fig. 2 the results for the static potential and its first excitation are shown. The ground state exhibits the well known linear rise, but clearly levels off at the breaking scale  $r_b$  to saturate at

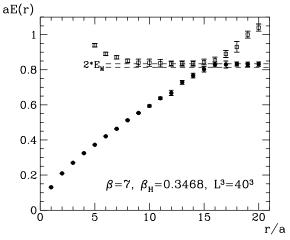


Figure 2. The potential and its first excited state. The dashed lines indicate twice the energy of the single meson state, as extracted from  $G_M(t)$ .

a constant value, which to very good accuracy agrees with twice the static-light meson energy. The energy of the first excited state quickly approaches the level of the MM state for  $r < r_b$ , but displays a linear rise for  $r > r_b$  in continuation of the ground state potential below  $r_b$ .

We note that a sharp crossover in the energy levels of the string and the MM state is also observed in a quenched calculation, which we have performed for comparison. In order to unambiguously establish string breaking it is necessary to analyse the composition of the energy eigenstates as a function of r and demonstrate the decay of the string by its coupling to the MM state. In Fig. 3 the maximum projection of the string and the MM operators onto the ground state of the potential are shown. For  $r < r_b$ , the ground state is clearly dominated by the string operator, the MM admixture is significantly smaller, albeit non-vanishing. This non-vanishing contribution survives in the quenched approximation, suggesting two possible sources for it: First, at small distances the two mesons may interact via gluon exchange. Second, the smeared scalar fields contain fuzzed links, which overlap at high smearing levels. While the former effect is physical the latter is a fuzzing artefact. Both lead to mixing with the physical string and decrease with growing r.

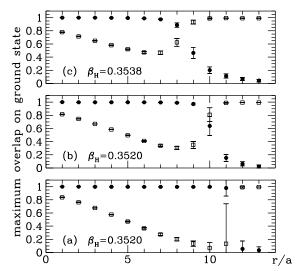


Figure 3. The maximum projections on the ground state of the string  $(a_{1,SS}, \text{ full circles})$  and the MM operator  $(a_{1,MM}, \text{ open squares})$  for two values of the hopping parameter and in quenched approximation (bottom) at  $\beta_G = 5.0$  on  $26^3$ .

In the dynamical case, there is a clear signal for mixing at  $r \simeq r_b$ , with comparable projections of both operator types onto the ground state. At  $r > r_b$  the potential is dominated by the MM operator, whereas now the first excited state consists of string contributions (not shown). Comparing with the quenched data, we note that the crossing in the ground state projection of the two operator types changes smoothly in the dynamical calculation, whereas it is a sharp step function in the quenched case (which is more pronounced at  $r > r_b$  where there are no fuzzing artefacts).

The properties of mixing can be studied by exploring different scalar masses (hopping parameters), c.f. Fig. 3. As expected, the string breaks earlier for lighter scalars (larger  $\beta_H$ ). In qualitative confirmation of [5] we also observe the mixing region to widen in this case, accompanied by a smoothing of the crossover in the potential.

Defining  $r_b$  by the interpolated distance with equal projections of string and MM operators,  $a_{1,SS}(r_b) - a_{1,MM}(r_b) = 0$ , the continuum approach in Table 1 yields  $r_b g^2 \approx 8.5$ , where  $m_G/g^2 = 1.60(4)$ ,  $m_S/g^2 = 0.839(15)$  are the

Table 1

$\beta_G$	$\beta_H$	L/a	$r_{ m b}/a$	$r_{ m b}g^2$
5.0	0.3520	36	$9.9 \pm 0.8$	$7.90 \pm 0.64$
7.0	0.3468	40	$14.8 \pm 1.0$	$8.47 \pm 0.57$
9.0	0.3438	52	$19.1 \pm 1.0$	$8.50 \pm 0.44$

lightest scalar glueball and meson states [7].

#### 4. CONCLUSIONS

We have presented clear numerical evidence for string breaking in the 3d SU(2) Higgs model. Wilson loops were demonstrated not to be suitable to extract the potential for  $r > r_b$ . Instead, string breaking may be studied via a mixing analysis of string and two-meson operators. The size of  $r_b$  and the width of the mixing region are functions of the mass of the dynamical particles. In a quenched calculation, only the crossover in the potential, but no smooth mixing region is observable. A similar anlaysis was also successful in the 4d SU(2) Higgs model [11], and the method may be expected to be applicable to QCD as well.

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